Additional Basic Signals

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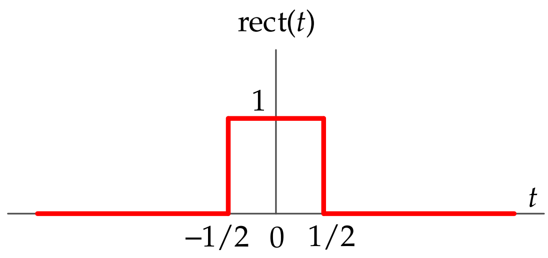
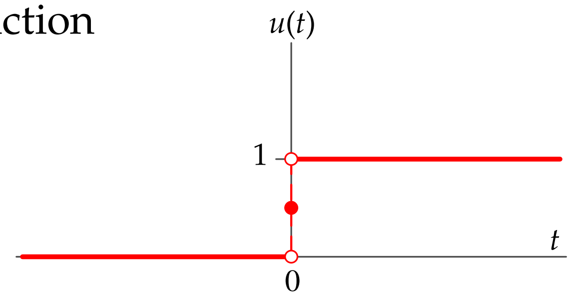


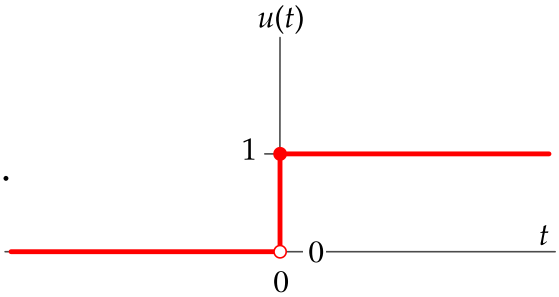
Unit step function or Heaviside step function (discontinuity at $t = 0$)

$$u(t) = H(t) = \begin{cases} 0, & t < 0 \\ 0.5, & t = 0 \\ 1, & t > 0 \end{cases}$$



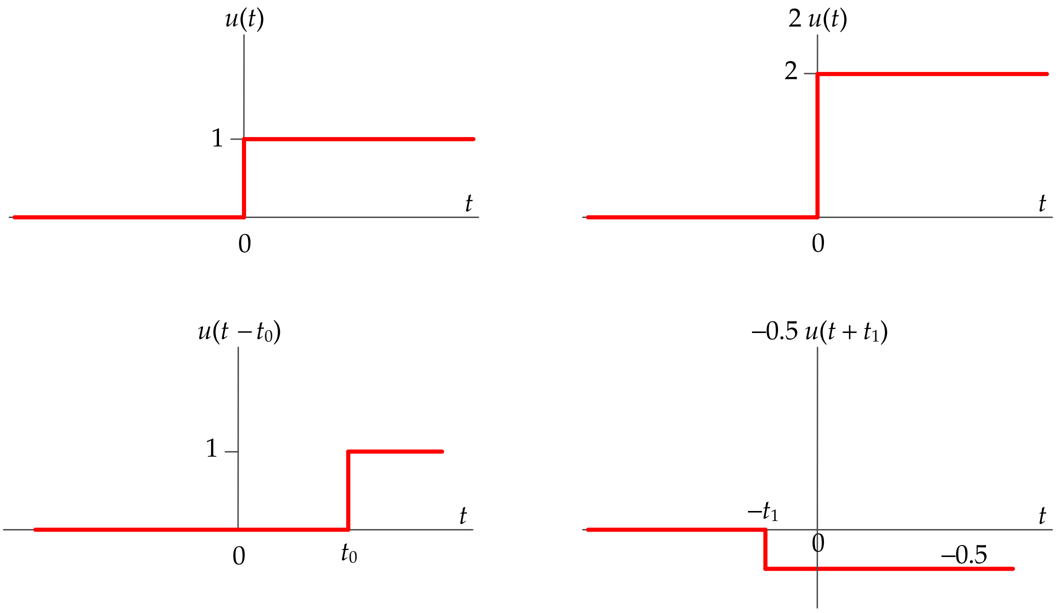
Or alternatively,

$$u(t) = H(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$



Alternatively, value at $t = 0$ is undefined. Any of these definitions is acceptable as this is not a problem we face in real-life.

Amplitude scaling and **time shifting** of the unit step function:

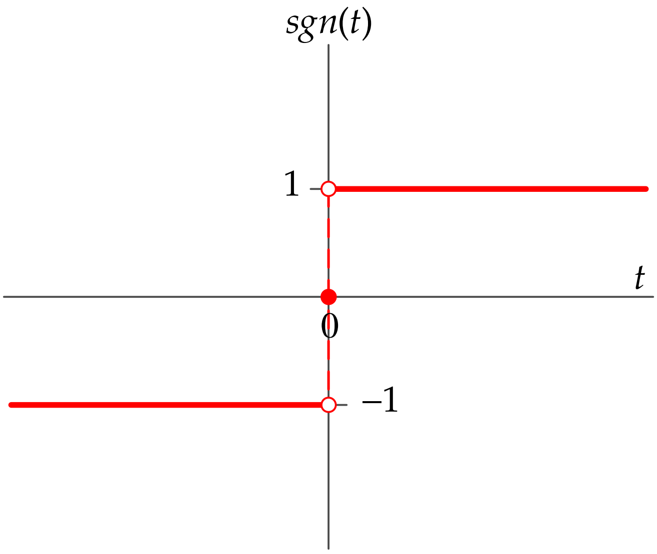


Sign function or signum function (sign of a real number)

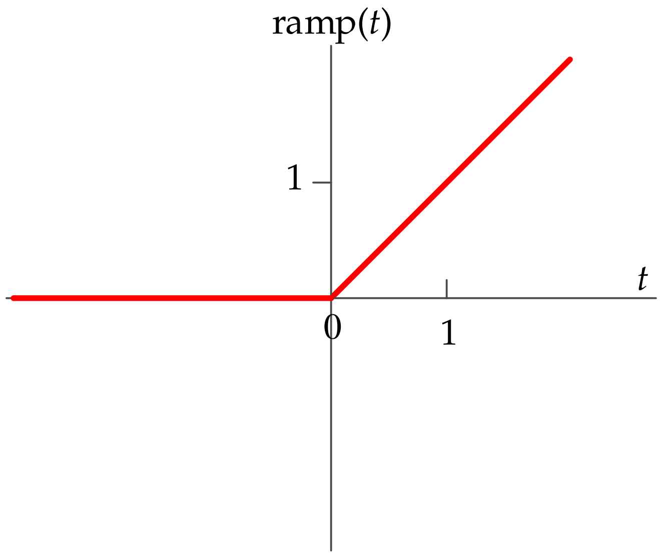
$$\text{sgn}(t) = \begin{cases} -1, & t < 0 \\ 0, & t = 0 \\ 1, & t > 0 \end{cases}$$

$$\text{sgn}(t) = \frac{t}{|t|} = \frac{|t|}{t}$$

$$\text{sgn}(t) = 2\,u(t) - 1$$



Unit ramp function:



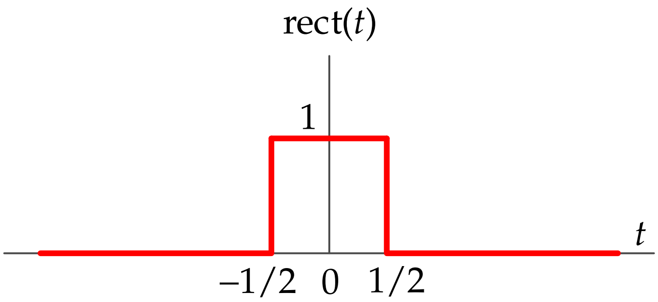
$$\text{ramp}(t) = r(t) = \begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases}$$

$$\text{ramp}(t) = t\,u(t)$$

$$\text{ramp}(t) = \int u(t)dt$$

$$u(t) = \frac{d}{dt}\text{ramp}(t)$$

Single rectangular pulse *or* unit rectangular pulse *or* rect (useful later to represent digital signals and square waves)

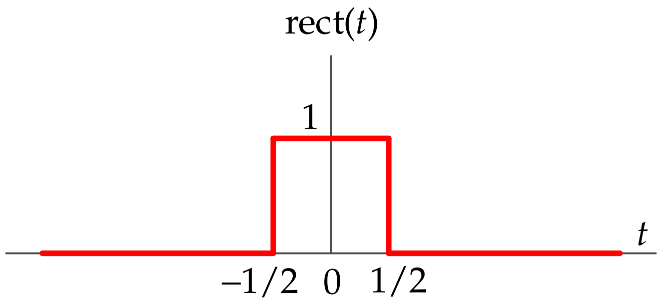


$$\text{Area} = W \times H = 1 \times 1 = 1$$

$$\text{rect}(t) = \Pi(t) = \begin{cases} 0, & t < -\frac{1}{2} \\ 1, & -\frac{1}{2} \leq t \leq \frac{1}{2} \\ 0, & t > \frac{1}{2} \end{cases} \quad \textit{or} \quad \text{rect}(t) = \begin{cases} 1, & |t| \leq \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

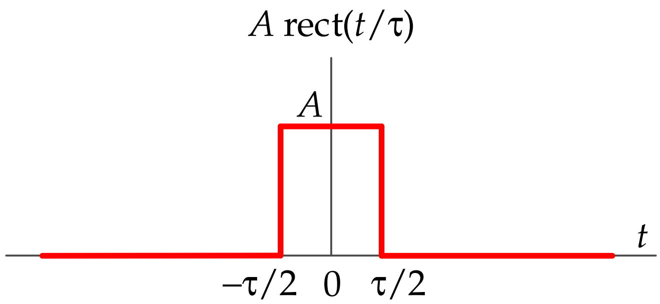
$$\text{rect}(t) = u\left(t + \frac{1}{2}\right) - u\left(t - \frac{1}{2}\right)$$

General rectangular pulse (notice **amplitude scaling** and **time spreading**)



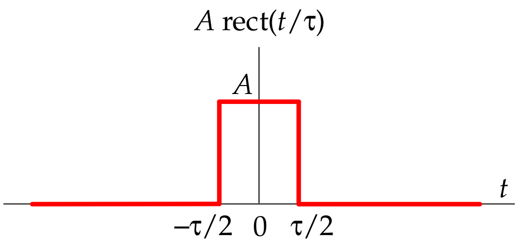
$$A\,\mathrm{rect}\left(\frac{t}{\tau}\right)=\begin{cases}A, & |t|\le\frac{\tau}{2}\\ 0, & \text{otherwise}\end{cases}$$

$$\text{Area}=W\times H=A\tau$$

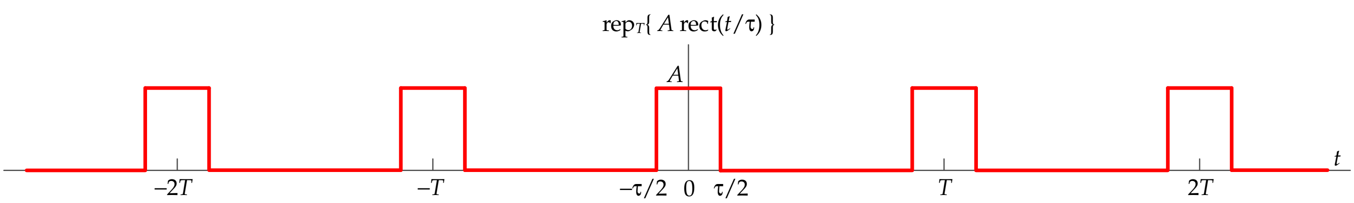


Repeated rectangular pulse

$$x(t)=\mathrm{rep}_T\left\{A\,\mathrm{rect}\left(\frac{t}{\tau}\right)\right\}$$



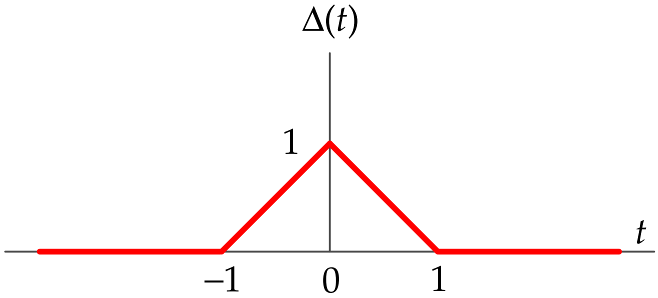
This $x(t)$ is now periodic function with **period,** T, in seconds



Single triangular pulse *or* unit triangular pulse

Notice the normalized area:

$$\text{Area} = \frac{1}{2} \times W \times H = \frac{1}{2} \times 2 \times 1 = 1$$

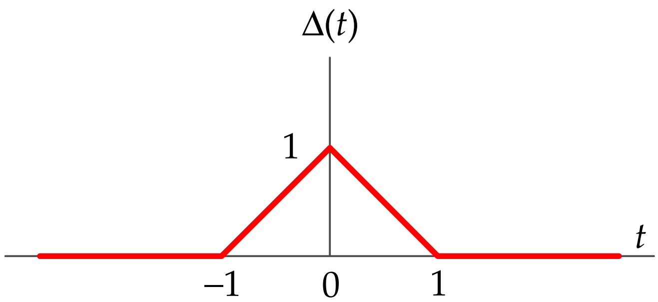


$$\text{tri}(t) = \Delta(t) = \begin{cases} 0, & t < -1 \\ t + 1, & -1 \le t < 0 \\ -t + 1, & 0 \le t \le 1 \\ 0, & t > 1 \end{cases}$$

or

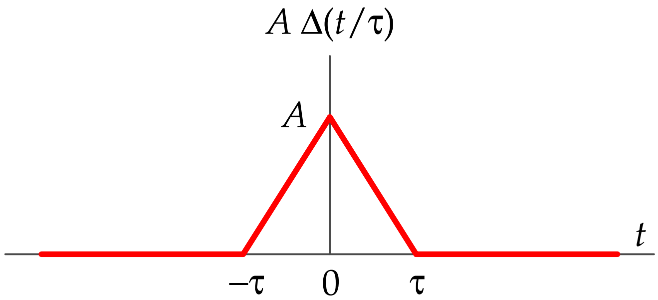
$$\Delta(t) = \begin{cases} 1 - |t|, & |t| \le 1 \\ 0, & |t| > 1 \end{cases}$$

General triangular pulse (notice **amplitude scaling** and **time spreading**)



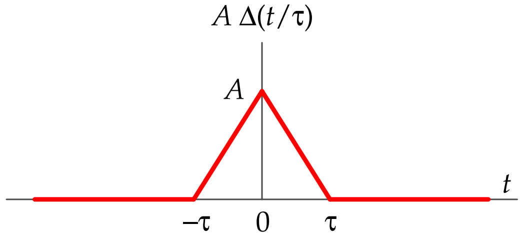
$$A\,\Delta\left(\frac{t}{\tau}\right) = \begin{cases} A - \left|\frac{A}{\tau} t\right|, & |t| \le \tau \\ 0, & |t| > \tau \end{cases}$$

$$\text{Area} = \frac{1}{2} \times W \times H = A\tau$$

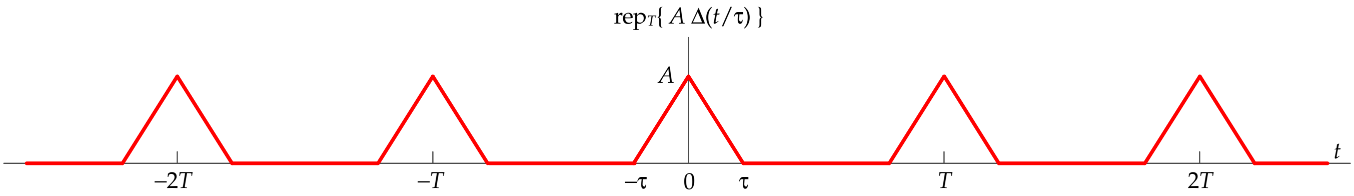


Repeated triangular pulse

$$x(t) = \text{rep}_T\left\{A\,\Delta\left(\frac{t}{\tau}\right)\right\}$$



This $x(t)$ is now periodic function with **period,** T, in seconds



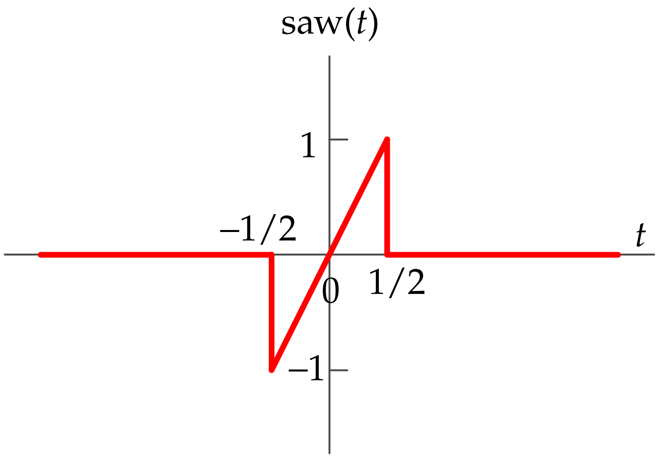
Single sawtooth pulse *or* unit sawtooth pulse

Notice the normalized area:

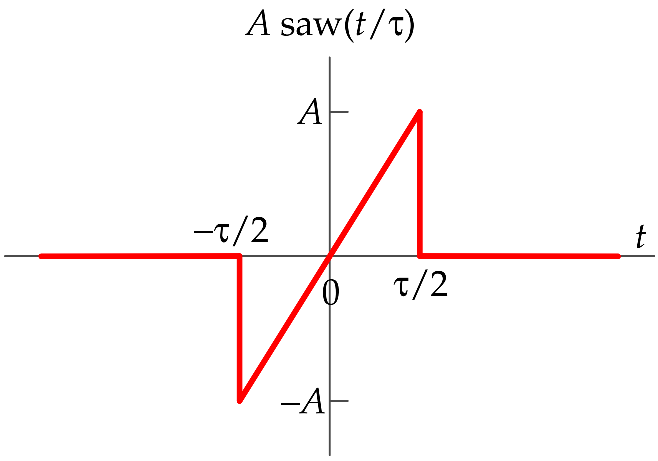
$$\text{Area} = 2 \times \frac{1}{2} \times W \times H = \frac{1}{2}$$

$$\text{saw}(t) = \begin{cases} 0, & t < -\frac{1}{2} \\ 2t, & -\frac{1}{2} \le t \le \frac{1}{2} \\ 0, & t > \frac{1}{2} \end{cases}$$

$$\text{saw}(t) = 2t \times \text{rect}(t)$$



General sawtooth pulse
(notice **amplitude scaling** and **time spreading**)

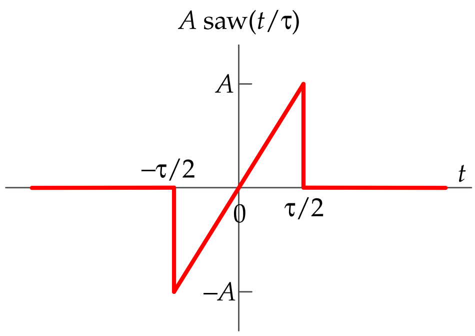


$$A\ \text{saw}\left(\frac{t}{\tau}\right) = \begin{cases} \frac{2A}{\tau} t, & |t| \le \frac{\tau}{2} \\ 0, & \text{otherwise} \end{cases}$$

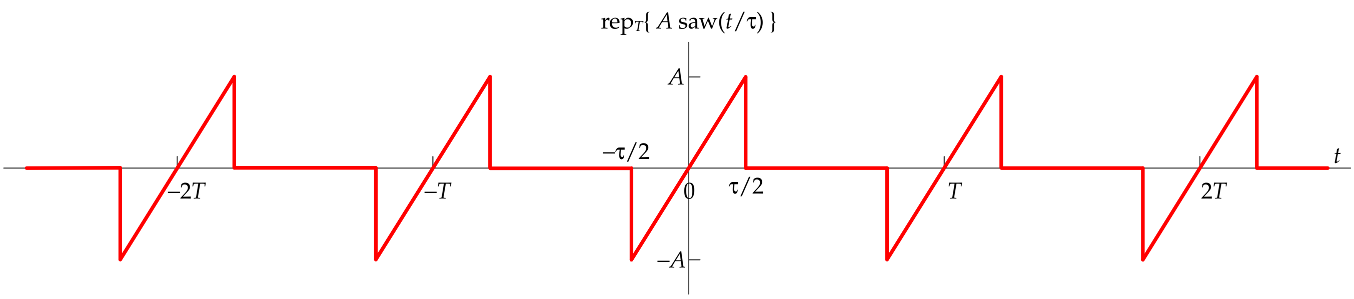
$$\text{Area} = 2 \times \frac{1}{2} \times W \times H = \frac{A\tau}{2}$$

Repeated sawtooth pulse

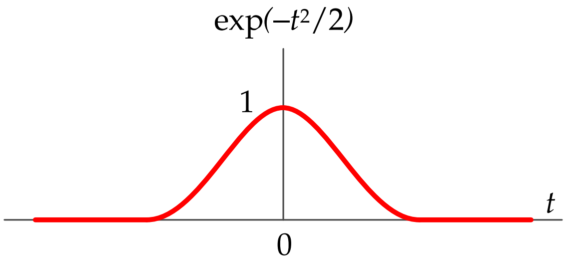
$$x(t) = \text{rep}_T\left\{A\ \text{saw}\left(\frac{t}{\tau}\right)\right\}$$



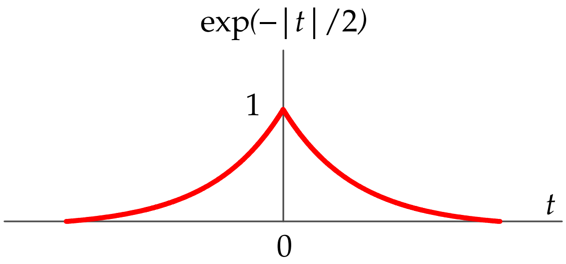
This $x(t)$ is now periodic function with **period,** T, in seconds



Other interesting pulses (that use exponential)



(Gaussian pulse)

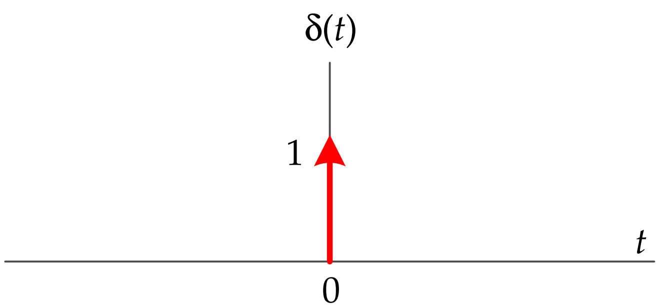


Impulse (Dirac delta function)

(ideal but *very* useful signal)

$$\delta(t) = \frac{d}{dt} u(t)$$

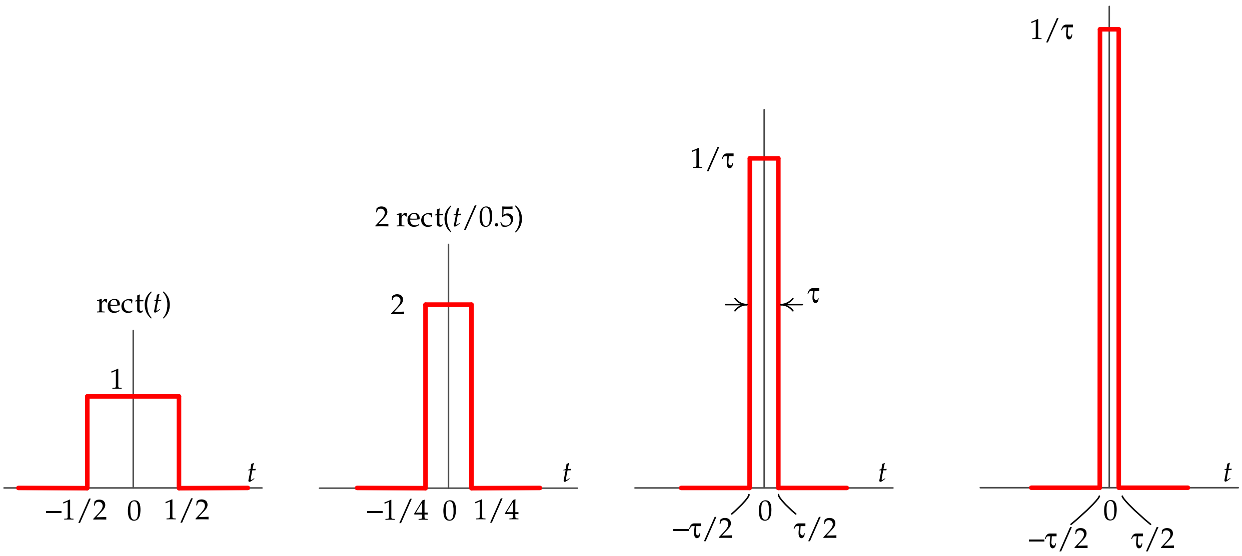
$$\int \delta(t) dt = u(t)$$



$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases}$$

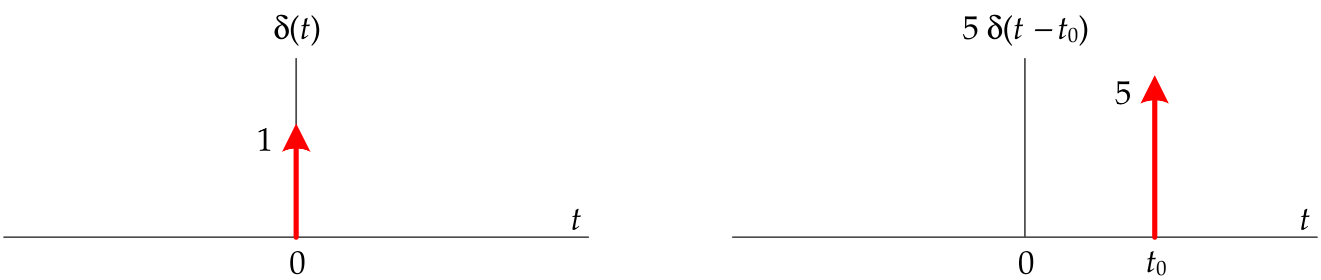
$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

$$\delta(t) = \lim_{\tau \to 0}\left(\frac{1}{\tau}\,\text{rect}\left(\frac{t}{\tau}\right)\right) \qquad \textit{or} \qquad \delta(t) = \lim_{\tau \to 0}\left(\frac{1}{\tau}\,\Delta\left(\frac{t}{\tau}\right)\right)$$



General impulse (notice **area scaling** and **time shifting**).

The amplitude remains infinity.



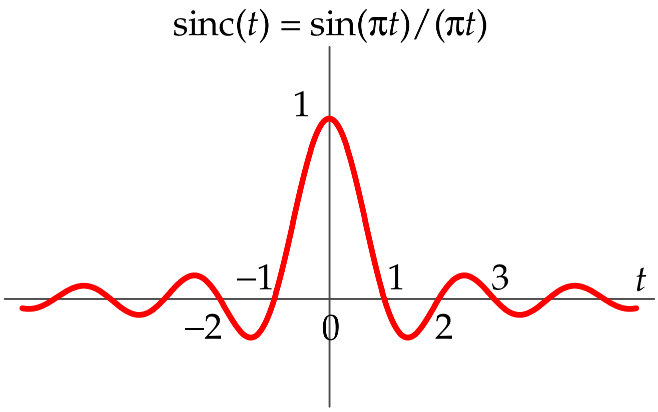
Sinc function

Two possible definitions

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\text{sinc}(t) = \text{Sa}(t) = \frac{\sin(t)}{t}$$

I will use the first definition (as seen in the figure).



To find the value of $\text{sinc}(t)$ at $t = 0$, use L'Hôpital's rule to find the limit of the $\sin(\pi t)/(\pi t)$ expression as $t \to 0$. For all other t instants, just calculate the $\sin(\pi t)$ value and divide it by the (πt) value.